Efficient P- and S- Wavefield Decomposition with Preserved Amplitude and Phase in Elastically Anisotropic Media

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Summary

We propose an efficient method for decomposing elastic wavefields into vectorized P-, SV-, and SH-wave modes in elastically anisotropic media while preserving amplitude and phase. Physics enhancement in wave propagation from acoustic to elastic in seismic imaging and inversion algorithms is increasingly important for providing a more complete subsurface information. However, this enhancement produces additional problems that need to be addressed for the correct application of the elastic algorithms. These are more susceptible to crosstalk artifacts from unmatched P/S modes, especially in anisotropic media where propagation and polarization directions differ. These artifacts can contaminate elastic reverse time migration and full-waveform inversion results. Existing decomposition methods commonly rely on simplifying assumptions, such as elliptical or weak anisotropy, or require substantial additional computational cost during wavefield extrapolation. Our method is derived from exact wave-mode expressions as functions of phase-propagation direction under the plane-wave solution, combined with phase-direction information, and therefore avoids restrictive anisotropic assumptions. Numerical tests using a 3D homogeneous TI model, the 2.5D SEAM Phase I model, and the 2.5D BP TI model demonstrate that the proposed method efficiently separates the total elastic wavefields into vectorized P/SV/SH wavefields with preserved amplitude and phase. The method can be incorporated into any existing anisotropic elastic modeling operators with minor additional computational cost and has strong potential to improve elastic imaging and inversion by reducing crosstalk artifacts and providing more accurate vector wavefields.

Introduction

Elastic seismic acquisition and imaging are increasingly important because they describe seismic propagation more accurately than acoustic methods and can better characterize subsurface structures and reservoir properties (e.g., Tarantola 1986, Singh et al. 2008, Liu et al. 2024, Macesanu et al. 2024, Liu et al. 2025). However, elastic wavefields contain multiple modes, and reliable separation of these modes remains a major challenge, particularly in anisotropic media. In such media, wave polarization depends on the elastic properties and local propagation direction, and the phase direction does not generally coincide with polarization direction. As a result, conventional P/S decomposition methods could produce mode leakage and crosstalk artifacts that degrade the quality of elastic imaging and inversion (e.g., Raknes and Arntsen 2017, Sun et al. 2025).

In isotropic media, Helmholtz decomposition separates a vector wavefield into curl-free P-wave and divergence-free S-wave components (Aki and Richards 2002). Extending this idea to anisotropic media is more challenging because wave polarization and propagation directions depend on the elastic stiffness tensor and vary with direction and spatial position. Dellinger and Etgen (1990) first extended the idea of P/S decomposition from isotropic to anisotropic media by computing particle-motion directions from the Christoffel equation in the wavenumber domain and constructing space-domain separation operators through inverse Fourier transform. Although this approach is theoretically rigorous, repeated Fourier transforms for each time step and location become computationally expensive for 3D heterogeneous models. Alkhalifah (2000) simplified anisotropic elastic wave propagation to an acoustic approximation by setting the shear velocity to zero and solving only for quasi-P waves, which is efficient, but it does not represent full elastic wave propagation. Liu et al. (2009) derived decoupled wave equations for P and SV waves in acoustic vertically transversely isotropic (VTI) media from the dispersion relation and solved the square-root operators using the optimized separable approximation (OSA) in the mixed space-wavenumber domain. While effective for P/S kinematics, this approach neglects mode conversion between P and S waves. Yan and Sava (2009), following Dellinger and Etgen (1990), addressed model inhomogeneity by convolving the wavefield with precomputed anisotropic spatial derivative operators derived from Christoffel-equation solutions, at each local grid. However, the memory and computational demands become very large for 3D applications. Cheng and Kang (2014, 2016) developed pseudo-pure P and S wave methods in TI media via two-cascaded steps. First, equivalent elastic anisotropic wavefields are simulated from a minimal second-order coupled system via a similarity transformation to the Christoffel matrix to obtain mode-enhanced wavefields. Subsequently, the pseudo-pure-mode wavefields are projected onto the polarization directions of quasi-P- or quasi-S waves to mitigate residual mode-leakage energy. These methods preserve kinematics well, but amplitude accuracy and mode conversion are not guaranteed.

At the same time, Cheng and Fomel (2014) formulated the anisotropic vector decomposition as space-wavenumber-domain operations in the form of Fourier integral operators and developed a fast implementation using low-rank approximations. Nevertheless, the low-rank approach remains costly for large 3D imaging problems. Zhu (2017) used the Helmholtz decomposition and a fast Poisson solver to reconstruct the separated scalar wavefield into the vectorized wavefields. However, this method is only discussed in isotropic media, and solving a Poisson problem would introduce additional computational complexity for each time step for imaging conditions. Yang et al. (2019) proposed a simplified decomposition operator, named pseudo-Helmholtz composition, with an assumption of elliptical anisotropy, and then reconstructed the vectorized wavefields via solving a Poisson problem. In practice, the elliptical anisotropy assumption is often violated, leading to mode leakage. Stovas et al. (2020) derived pure-mode equations for P- and S- waves in TI media without weak anisotropy assumptions, from dispersion analysis, but their formulation focuses primarily on kinematics rather than amplitude fidelity and mode conversion.

More recent studies employ data-driven approaches, such as deep learning with generative adversarial networks (GANs) by Kaur et al. (2021), although issues of training, generalization, and extension to 3D may require further testing. Wang and Zhang (2022) proposed a fast 2D method based on Christoffel equations and Poynting vector, to achieve scalar quasi-P and quasi-S wavefields for 2D cases, and also provide the angle correction for the tilted TI media. Zhang et al. (2022) derived an anisotropic Helmholtz-type decomposition with first-order Taylor expansion and weak anisotropy, but their method also requires multiple Fourier transforms. Zhang et al. (2023a), Zhang et al. (2023b) developed another faster pseudo-Helmholtz decomposition method under the elliptical assumption. In numerical testing, their method still could generate mode leakages (Fan et al. 2024). Fan et al. (2024) propose a method to decompose the elastic wavefields in 2D VTI media by introducing scalar operators to transfer operations from the mixed space-wavenumber domain to the space domain and the local wave propagation direction from the Poynting vector. Yao et al. (2024) proposed pseudo-Helmholtz decomposition for an elastic VTI wavefield based on wavefront phase direction under first-order Taylor approximation on the eigenvalues and eigenvectors of the Christoffel matrix. Both of their methods still involve approximation.

To address the challenge of efficient P/S wave decomposition in anisotropic media while preserving amplitude, phase, and vector information without restrictive anisotropic assumptions, we propose a new method based on exact solutions of the decomposed wavefields and phase Poynting vectors. We demonstrate the feasibility of the method using a 3D homogeneous anisotropic model, the 2.5D SEAM Phase I model with constant anisotropic parameters, and the 2.5D BP TI model with spatially varying anisotropy. We then discuss its performance and implications for elastic imaging and inversion.

Method

Starting from Hooke's law and Newton's second law:

$$\rho \dot{v}_i = \sigma_{ij,j} + \boldsymbol{f_s},$$
$$\dot{\sigma}_{ij} = c_{ijkl} v_{k,l} \qquad (1)$$

where σ_{ij} is the stress tensor, c_{ijkl} is the elastic stiffness tensor, v_i is the particle velocity, ρ is density, $\boldsymbol{f_s}$ is the source term, and i, j follow Einstein notation.
For a VTI medium, the stiffness matrix is

$$\mathbf{C(VTI)} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \qquad (2)$$

with
$c_{33} = \rho v_{p_0}^2$, $c_{11} = (2\varepsilon + 1)c_{33}$, $c_{44} = \rho v_{s_0}^2$,
$c_{66} = (2\gamma + 1)c_{44}$, $c_{12} = c_{11} - 2c_{66}$,
$c_{13} = \rho\sqrt{2\delta v_{p_0}^2\left(v_{p_0}^2 - v_{s_0}^2\right) + \left(v_{p_0}^2 - v_{s_0}^2\right)^2} - \rho v_{s_0}^2$, where ε, γ and δ are Thomsen parameters, v_{p_0} and v_{s_0} are the P and S wave propagation velocity along the symmetry axis (Thomsen 1986). Here, Thomsen parameters are used to parameterize the TI medium, but the proposed method itself does not assume weak anisotropy.
Under a plane-wave assumption and neglecting spatial derivatives of model parameters in locally smooth media, the Christoffel matrix can be written as:

$$\mathbf{M(k)} = \begin{pmatrix} {k_x}^2 v_p^2(1+2\epsilon) + k_y^2 v_s^2(1+2\gamma) + k_z^2 v_s^2 & k_x k_y\left(-v_s^2(1+2\gamma) + v_p^2(1+2\epsilon)\right) & k_x k_z\sqrt{\left(v_p^2 - v_s^2\right)\left(v_p^2(1+2\delta) - v_s^2\right)} \\ k_x k_y\left(v_p^2(1+2\epsilon) - v_s^2(1+2\gamma)\right) & k_x^2 v_s^2(1+2\gamma) + k_y^2 v_p^2(1+2\epsilon) + k_z^2 v_s^2 & k_y k_z\sqrt{\left(v_p^2 - v_s^2\right)\left(v_p^2(1+2\delta) - v_s^2\right)} \\ k_x k_z\sqrt{\left(v_p^2 - v_s^2\right)\left(v_p^2(1+2\delta) - v_s^2\right)} & k_y k_z\sqrt{\left(v_p^2 - v_s^2\right)\left(v_p^2(1+2\delta) - v_s^2\right)} & k_x^2 v_s^2 + k_y^2 v_s^2 + k_z^2 v_p^2 \end{pmatrix} \qquad (3)$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the phase-wavenumber vector. Its eigen decomposition is

$$\mathbf{M} = \mathbf{V\Lambda V^{-1}}, \qquad (4)$$

where the $\mathbf{\Lambda}$ contains the eigenvalues associated with the P, SV, and SH modes, and the columns of $\mathbf{V}$ are corresponding polarization vectors. These polarization vectors specify the exact directions of particle motion for each wave mode as functions of the phase-propagation direction and the local elastic stiffness at each numerical grid point.
We solve the 3D anisotropic elastic wave propagation on the staggered grid (e.g., Virieux 1986, Graves 1996), using the Devito framework (Louboutin et al. 2019, Luporini et al. 2020). The decomposition is then performed by projecting the total vector wavefield onto the exact polarization directions derived from the Christoffel matrix and combining this information with the phase Poynting-vector direction. In this way, the method preserves both amplitude and phase of the separated P, SV, and SH wavefields while avoiding the large memory and computational burden of repeated Fourier-domain or Poisson-based operations. The

overall workflow of the proposed decomposition method is shown in Figure 1.

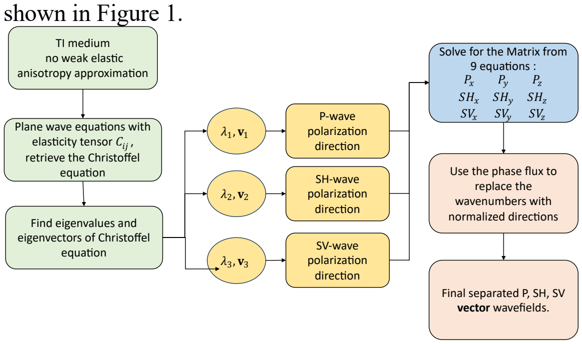

Figure 1. Schematic workflow of the proposed anisotropic elastic wavefield decomposition method.

Numerical examples

3D homogeneous model

We first evaluate the proposed method using a 3D homogeneous TI model with constant anisotropic parameters to verify decomposition accuracy and preservation of amplitude and phase (Figure 2). Although the model is simple, it provides an important validation case because anisotropic mode separation can be examined in isolation from complications associated with structural heterogeneity, lateral parameter variation, and scatterings. In such a controlled experiment, the exact polarization directions are determined solely by the local anisotropic properties and propagation direction, making this example an effective test of whether the proposed method correctly honors anisotropic wave physics. The use of an explosive source together with a vertical dipole source generates coupled P and SV wavefields, allowing the separated quasi-P, quasi-SV, and quasi-SH modes to be evaluated simultaneously. The comparison with Helmholtz decomposition highlights the limitations of isotropic-based separation in anisotropic media, where mode leakage remains visible. In contrast, the proposed method produces cleaner separated wavefields with preserved amplitude and phase, demonstrating its ability to capture the correct vector-wavefield behavior in TI media.

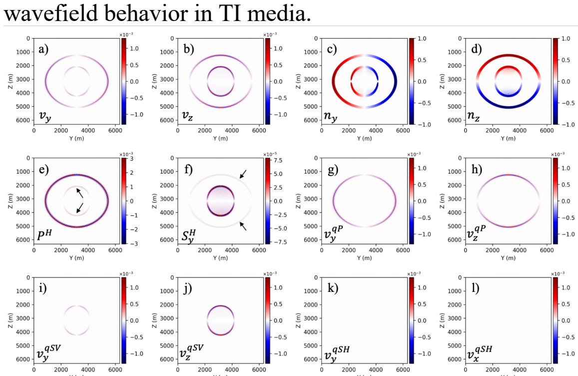

Figure 2. Results for 3D homogenous model. In this model, the model parameters are $\epsilon = 0.2$, $\delta = 0.1$, $\gamma = 0.2$, $v_p = 3km/s$, $v_s = 2km/s$ and $\rho = 1g/cm^3$. The source is an explosive source plus a vertical dipole source. (a) and (b) show the total particle-velocity wavefields in the y- and z-directions. (c) and (d) show the normalized phase-direction components n_y and n_z estimated from the phase Poynting vector. (e) and (f) show P-wavefield and y component of S-wavefield, separated by Helmholtz decomposition, where strong P-wave and S-wave leakage artifacts (black arrows) are visible. (g), (h), (i), and (j) show the separated quasi-P and quasi-SV wavefields in the y- and z-directions using the proposed method. (k) and (l) show y- and x- directions of the quasi-SH wavefield components, which should vanish in this example.

2.5D SEAM phase I model

We next test the proposed method on the 2.5D SEAM Phase I model (e.g., Fehler and Larner 2008) with constant anisotropic parameters of $\epsilon = 0.2$, $\delta = 0.1$, and $\gamma = 0.15$, as shown in Figure 3. This model introduces significant structural complexity and therefore provides a more realistic benchmark for evaluating decomposition quality in anisotropic media. The results show that, compared with Helmholtz decomposition, the proposed method more effectively suppresses mode leakage and preserves the amplitude and phase of the separated vector wavefields.

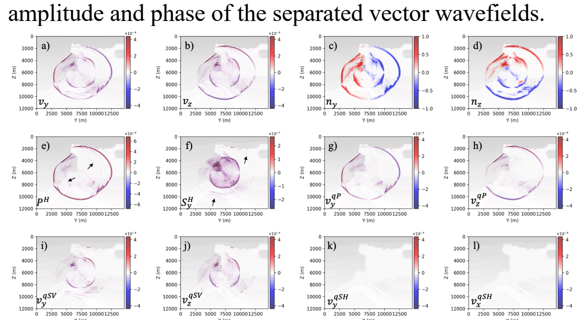

Figure 3. Results for the 2.5D SEAM Phase I model with constant anisotropic parameters of $\epsilon = 0.2$, $\delta = 0.1$, and $\gamma = 0.15$. This 3D model is constructed by laterally extending the 2D elastic SEAM Phase I model. The panel layout is the same as in Figure 2. Mode-leakage artifacts remain visible in the Helmholtz-decomposed wavefields, as indicated by the black arrows. In contrast, the proposed method yields cleaner quasi-P and quasi-SV wavefields with substantially reduced leakage.

2.5D BP TI model

We further test the proposed method on the 2.5D BP TI model (Shah 2007), which represents a more realistic heterogeneous anisotropic medium (Figure 4). In this example, the S-wave velocity is estimated using a constant ratio of $v_p/v_s = 2$, and density is defined by $\rho = 0.404\, v_p + 0.395$. Compared with the previous examples, this model includes stronger lateral and vertical variations in velocity and anisotropic structure, making wave propagation and mode separation more challenging. This example is therefore used to evaluate the robustness of the proposed method in a geologically complicated TI setting. The results show that conventional Helmholtz decomposition still exhibits noticeable mode leakage, although much weaker than in the SEAM example, due to the explosive source in the water. In contrast, the proposed method effectively separates the quasi-P and quasi-SV wavefields while preserving amplitude and phase and directly provides vectorized wavefields. These results further demonstrate the applicability of the method to realistic anisotropic models used in elastic imaging and inversion.

Discussions and Conclusion

We developed an approach to separate the total elastic response into vectorized P-, SV-, and SH-wavefields rather than only scalar wavefields. Our approach does not require elliptical-anisotropy or weak-anisotropy assumptions and is capable of preserving amplitudes and phases. The method is based on exact polarization solutions of the Christoffel matrix as functions of phase-propagation direction and elastic stiffness. Therefore, it is more general than pseudo-Helmholtz-type methods derived under approximate anisotropic conditions. In addition, because the decomposition is formulated as an add-on to existing elastic modeling operators, the additional computational cost is modest compared with approaches that require repeated Fourier transforms, large pre-computed convolution operators, or Poisson solves at every time step. We validated our method through numerical examples using a 3D homogeneous TI model, the 2.5D SEAM Phase I model, and the 2.5D BP TI model. Results demonstrate that the method accurately separates the total elastic wavefield into vectorized wave modes while suppressing mode leakage compared with existing approaches.

Although the present work focuses on TI media, the method can be readily extended to TTI media by performing local coordinate transformations that align the stiffness tensor with the tilted symmetry axis. The decomposed local wavefields can then be rotated back into the global coordinate system. This extension offers a practical pathway for applying the method in more general anisotropic models. These separated wavefields can be directly used to reduce P/S crosstalk artifacts and to construct more advanced imaging conditions, such as angle-domain or mode-dependent imaging conditions (e.g., Hu et al. 2023a, Hu et al. 2023b). The method is therefore well-suited for elastic reverse time migration, mode-dependent imaging conditions, and elastic full-waveform inversion.

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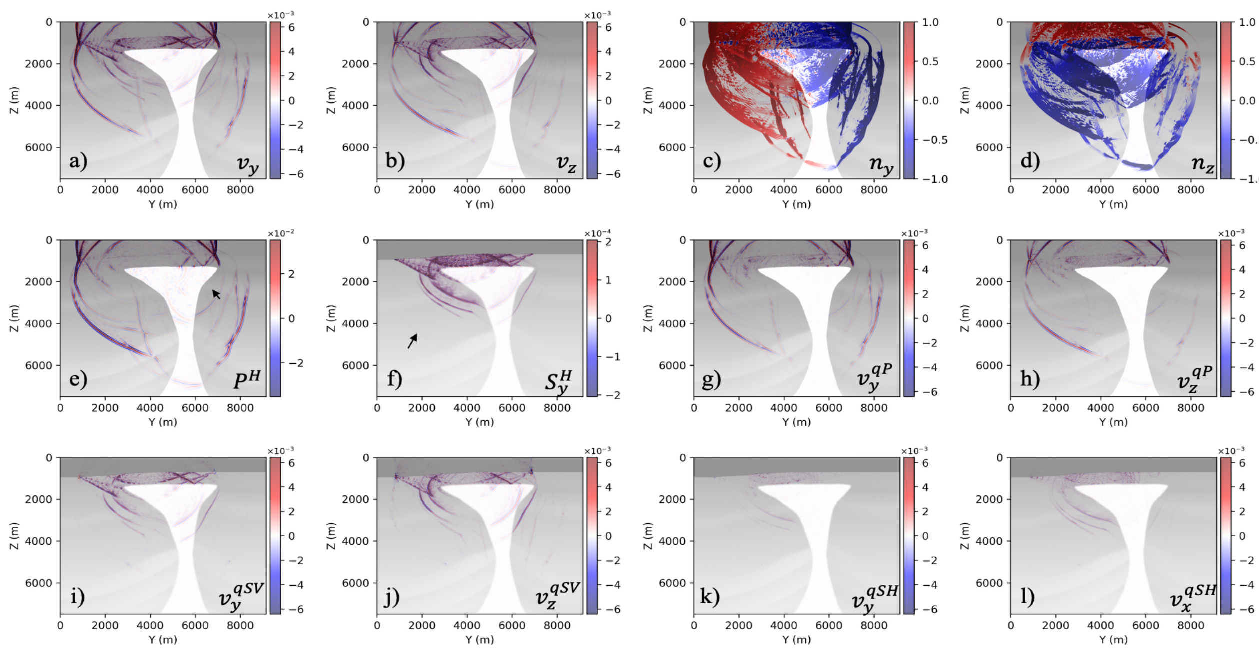

Figure 4. Results for the 2.5D BP TI model. The panel layout is the same as in Figure 2. Slight mode leakages remain visible in the Helmholtz-decomposed wavefields due to the weaker converted S waves from the explosive source in the water. The proposed method, however, provides cleaner quasi-P and quasi-SV wavefields and preserves the vector nature of the separated wavefields in this complicated heterogeneous TI model.