Incorporating Geologic Information into Reflection Tomography with a Dip Oriented Gaussian Filter

C. Zhou* (Petroleum Geo-Services)

SUMMARY

Reflection tomography is non-unique and a regularization term is usually added into its objective function as an
additional constraint. The anisotropic Gaussian filter has been successfully employed as such a regularization operator.
By orienting the smoothing axes along the local dip directions, the new Gaussian filter helps reflection tomography
produce models that conform to the reflector structures. For efficiency purpose, the 3D Gaussian filter is factorized
into three 1D filters in a non-orthogonal coordinate system. The dip oriented Gaussian filter also provides the freedom
for applying spatially variable smoothing.

Introduction

The non-uniqueness of reflection tomography requires additional constraints in order to produce desired models for depth migration. One way to add such constraints is the Tikhonov Regularization (Tikhonov and Arsenin, 1977) that incorporates a roughening operator into the tomography equation system. Harlan (1995) reparameterizes the regularization by using a smoothing operator to replace the roughening operator. The reparameterized operation is also referred to as preconditioning (Clapp et al. 2004). In many cases, a model that follows the earth geology is desired and therefore, geologic information is needed as a constraint. Clapp et al. (2004) propose a steering filter that uses the dip information to constrain reflection tomography. Such a steering filter is also employed by Bakulin et al. (2010) for anisotropic tomography with well data. In addition to producing a geologically plausible model, geologic constraint also improves the speed of convergence of tomography (Clapp et al., 2004).

Following Harlan's reparameterization, Zhou et al. (2009) utilize an anisotropic Gaussian filter as the smoothing operator to control the model smoothness in each Cartesian axis direction. An appealing feature of the Gaussian filter is that it provides quantitative controls of model smoothness in each axis direction. Very often, it is desired that the axes of Gaussian smoothing conform with the actual geology, rather than with the Cartesian inline, cross line and depth directions. In this paper, we extend the anisotropic Gaussian filter by orienting the smoothing axes according to the local dip information in order to build a geologically plausible model.

The dip oriented Gaussian filter

The 3D dip oriented anisotropic Gaussian filter (Fig. 1) is given by

$$g(\sigma_u,\sigma_v,\sigma_w)=\frac{1}{(2\pi)^{3/2}\sigma_u\sigma_v\sigma_w}\exp\left\{-\frac{1}{2}\left(\frac{u^2}{\sigma_u^2}+\frac{v^2}{\sigma_v^2}+\frac{w^2}{\sigma_w^2}\right)\right\}, \tag{1}$$

with σ_u, σ_v and σ_w being the Gaussian standard deviations.

The local coordinate system (*u, v, w*) is oriented by rotating the (*x, y, z*) coordinate system about the z axis by azimuth angle α followed by another rotation about the y axis by the dip angle θ. The rotation matrices are

$$R_z=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}, \tag{2}$$

$$R_y=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}. \tag{3}$$

Although this 3D filter can be directly implemented by replacing *u, v* and *w* with *x, y* and *z,* it is extremely inefficient. For each output data point, the cost is a three dimensional convolution. A better way is to separate this 3D filter into three cascaded 1D Gaussian filters (Lampert and Wirjadi, 2006a):

$$g(\sigma_u,\sigma_v,\sigma_w)=g(\sigma_v)g(\sigma_u)g(\sigma_w), \tag{4}$$

$$g(\sigma_t)=\frac{1}{\sqrt{2\pi}\sigma_t}\exp\left\{-\frac{1}{2}\left(\frac{t^2}{\sigma_t^2}\right)\right\}, \tag{5}$$

where t stands for *u, v* or *w*. Three cascaded 1D filter operations are more efficient than a 3D operation. Because the orientation is spatially variable, existing fast implementations, such as the recursive implementation (Yong and van Vliet, 1995), are impractical. Direct convolution is the preferred option. If the filtering process is performed in order of $g(\sigma_w)$, $g(\sigma_u)$ and $g(\sigma_v)$, for each output data point, a line of data along grid v are needed to perform $g(\sigma_v)$. For each data point on this grid line, a line of data along grid u have to be processed by operation $g(\sigma_u)$, which makes $g(\sigma_u)$ perform filtering on a plane of data. Accordingly, $g(\sigma_w)$ has to be conducted on a 3D volume of data. Each data point in this 3D volume on (u, v, w) grids has to be interpolated in three dimensional fashion from input data on (x, y, z) grids, which may be costly.

The 3D filter defined in equation (1) can also be factorized into three cascaded 1D filters on a non-orthogonal coordinate system (Lampert and Wirjadi, 2006a; Wirjadi and Breuel, 2005), of which one axis is chosen to be the global x, y, or z axis. The first filtering operation is done on the data grid along this global axis and thus no interpolation is needed. The subsequent two filtering operations along the other two non-orthogonal axes, however, do require interpolation but only for one plane compared to a 3D volume. The trade-off is that, depending on the interpolation algorithm, normally the first filtering operation is needed to be conducted for two planes in order to produce a plane of data for the second filtering operation. As Lampert and Wirjadi (2006b) indicate, factorizing the filter onto a non-orthogonal coordinate system yields a more efficient filter. Therefore, it is chosen for the implementation of the dip oriented Gaussian filter.

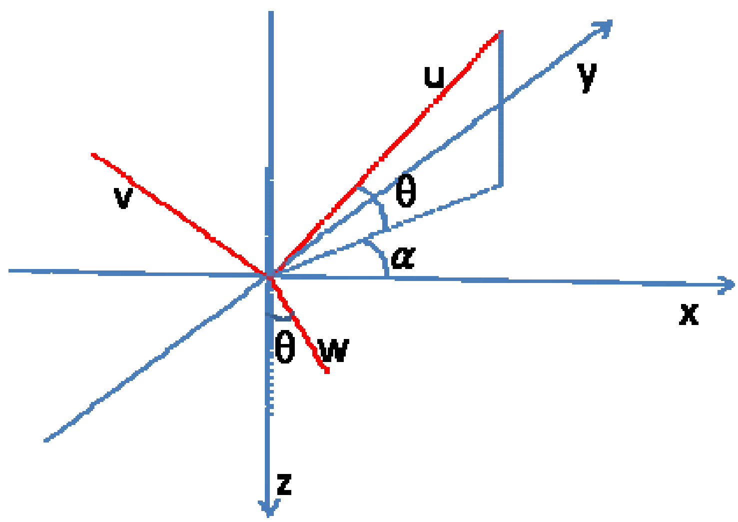

Figure 1 *The local dip orientation and global coordinate system.*

To test the dip oriented Gaussian filter, we convolve the filter with a 500 x 500 model that has the value of 1.0 at (250,250) and zeros elsewhere. The result for a constant dip field with a dip angle of 27 degrees is shown in Fig. 2a while Fig. 2b displays the filtered model when the dip angle changes from -45 degrees to 45 degrees linearly from left to right.

Reflection tomography with the dip oriented Gaussian smoothing

The regularized reflection tomography equation (Zhou et al., 2009) can be expressed as

$$\mathbf{G}\mathbf{G}^T\mathbf{A}^T\mathbf{A}\Delta\mathbf{s}+\tau\Delta\mathbf{s} = \mathbf{G}\mathbf{G}^T\mathbf{A}^T\mathbf{b}, \quad (6)$$

where $\mathbf{A}$ is the linear tomography operator, $\mathbf{T}$ stands for transpose, $\mathbf{b}$ is the data vector that contains the travel time residuals, τ is the regularization factor (also called the trade-off factor), $\Delta\mathbf{s}$ is the slowness update and $\mathbf{G}$ is the anisotropic 3D Gaussian filter. If the regularization is imposed on the desired slowness $\mathbf{s}$, rather than on the slowness update, equation (6) is adjusted as

$$\mathbf{G}\mathbf{G}^T\mathbf{A}^T\mathbf{A}\Delta\mathbf{s}+\tau\Delta\mathbf{s} = \mathbf{G}\mathbf{G}^T\mathbf{A}^T\mathbf{b} - \tau\mathbf{s}_0, \quad (7)$$

with $\mathbf{s} = \mathbf{s}_0 + \Delta\mathbf{s}$ where $\mathbf{s}_0$ is the starting slowness. By supplying equation (6) or (7) with the dip oriented Gaussian filter, we develop a tomography equation system that builds models conforming to the earth geology.

A dip field is needed in order to guide the dip oriented Gaussian filter. The dip information at a point can be represented by a 3D vector or by azimuth angle and dip angle. Another convenient paramerization is using the inline slope and cross line slope. The dip information can be either automatically scanned from the migration stack or calculated from picked horizons. The former requires no human interference other than providing proper parameters while the latter lets users have more control. The continuous tomography strategy described in Zhou et al. (2009) still applies to the new tomography system to keep the tomography process from getting into a premature local minimum. The tomography process starts with big standard deviations for the first iteration to obtain a smooth model and then gradually reduces them in subsequent iterations for finer scale details.

A velocity model that is built by tomography regularized by the dip oriented Gaussian filter is overlaid on its corresponding seismic stack and shown in Fig. 4. Because geologic information is used in the tomography process, the model conforms to the reflector structure in the stack. Due to the fact that the smoothing directions are spatially variable, a direct convolution filter replaces the usual recursive filter for computation efficiency. This results in a slow smoothing operation. The computation cost is proportional to the length of the filter that is controlled by its standard deviation. Moreover, the filter operation involves interpolation, which also incurs extra computation. Nevertheless, because each filter is independent, it provides the freedom for applying spatially variable smoothing.

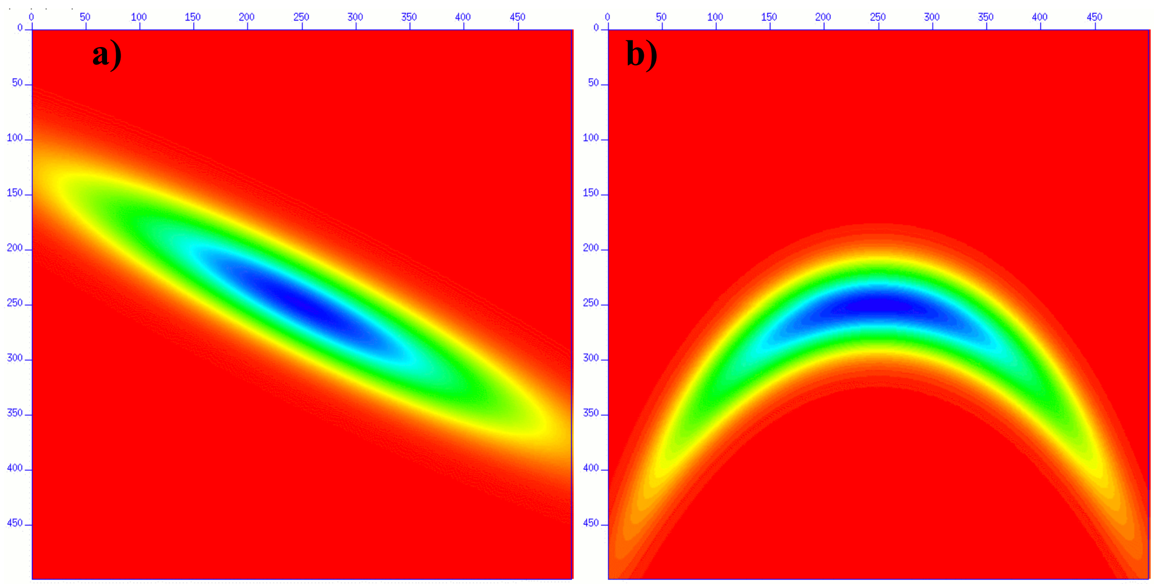

Figure 2 *Filtering a 500 x500 mode l that has 1.0 at (250,250) and zeros elsewhere with dip oriented Gaussian filters: a) the dip field is constant (27 degrees); b) the dip field linearly changes from -45 degrees to 45 degrees from left to right.*

Conclusions

We have described an approach to regularization for reflection velocity tomography using the dip oriented Gaussian filter. A dip field obtained from the migration stack is used to orient the filters. This approach yields geologically plausible models but it incurs more computation cost due to variable smoothing directions. Because each filter is independent, it provides the freedom for smoothing along spatially variable directions with spatially variable smoothing lengths.

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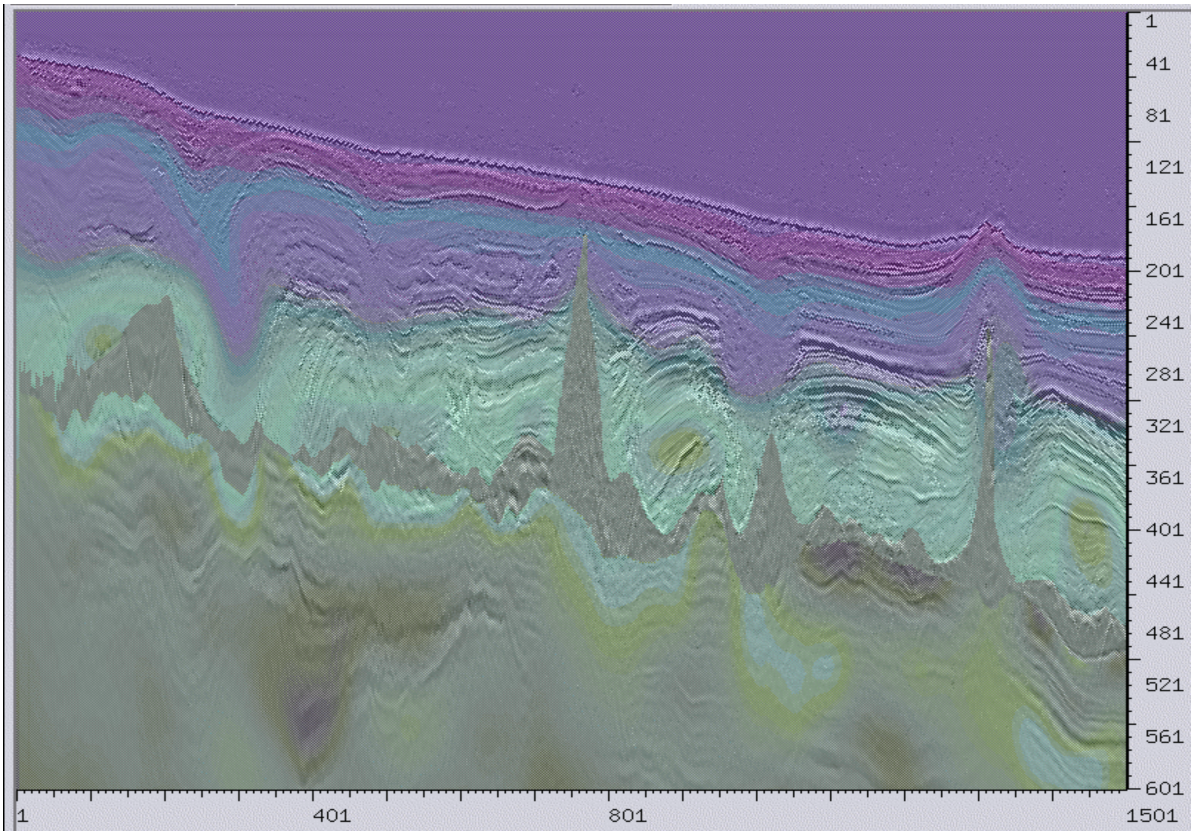

Figure 3 *The velocity model, built with the dip information incorporated, overlaid on the stack.*

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