

High Resolution Imaging by Wave Equation Reflectivity Inversion

A. Valenciano* (Petroleum Geo-Services), S. Lu (Petroleum Geo-Services),
N. Chemingui (Petroleum Geo-Services) & J. Yang (University of Houston)

SUMMARY

We discuss a wave equation solution for broadband imaging of seismic data. The new methodology poses depth migration as a wave equation reflectivity inversion; enhancing the resolution and balancing the wave number content of the depth migrated images. The method explicitly computes the point spread functions (PSF) by wave equation modelling and migration of discretely sampled point scatterers. The reflectivity function is recovered by solving a linear system of equations that deconvolves the multidimensional PSF from the migrated image. The implementation is efficient and effectively incorporates the spatial variability of the PSF. Results from synthetic and field data examples demonstrate the improvements in spatial resolution after the wave equation reflectivity inversion.

Introduction

Seismic modelling/migration operators are non-unitary (Claerbout, 1992). They produce a blurred representation of the earth reflectivity, with limited wave number content and resolution (Gelius et al., 2002, Lecomte, 2008). The image resolution at a given depth is controlled by the acquisition parameters (source signature, frequency bandwidth, acquisition geometry), the earth properties at the reflector depth, and the overburden (velocity, illumination, and attenuation). Recent efforts to improve resolution have produced new acquisition solutions that allow proper de-ghosting during time processing (e.g. Carlson et al., 2007). These solutions generate data with broader bandwidth, and enable removal of notches from the frequency spectra. However they do not compensate for heterogeneities in the earth and/or limitations of the acquisition geometry that could reduce the wave number content of the depth migrated images.

It is possible to improve the wavenumber content of the migration by posing the imaging problem in terms of reflectivity inversion (Nemeth et al., 1999, Prucha and Biondi, 2002, Valenciano et al., 2006). Two main approaches to solve the reflectivity inversion problem have been discussed in the literature, both based on least squares. In the first approach, the point spread function (PSF) or Hessian is explicitly computed. This can be performed with various degrees of approximation and computational complexity (Hu et al., 2001, Rickett, 2003, Plessix and Mulder, 2004, Guitton, 2004, Valenciano et al., 2006). Then one solves for the unknown reflectivity by solving a linear system of equations (multidimensional deconvolution) where the PSF and the migrated images are known. The second approach is computationally more challenging; it computes least squares migration images by means of data space residual reduction in an iterative fashion (Nemeth et al., 1999, Prucha and Biondi, 2002). The computational cost of this method depends on the number of iterations.

Here, we propose a wave-equation least-squares reflectivity inversion (WEI) that enhances the wave number content of the depth migrated images. Our method explicitly computes the PSF by wave-equation modeling/migration using a set of point scatterers distributed through the model space. It assumes a degree of stationarity of the PSF as they are later interpolated in order to have a contribution per image point. The final step is to solve a linear system, by using a conjugate gradient solver, where the migrated images and the PSF are the known quantities and the reflectivity is unknown (Valenciano et al., 2006). Results from a synthetic Sigsbee2A model and a dual-sensor field dataset from the North Sea demonstrate the image wave number content improvements resulting from applying WEI.

Wave-equation reflectivity inversion (WEI)

Least-squares inversion provides a theoretical framework to compensate images for limitations in: the acquisition geometry, complexities at the reflector depth, and earth properties in the overburden. The algorithm can be summarized as follows by using the Born-modelling framework. Given a linear wave equation modelling operator \mathbf{L} , compute synthetic data \mathbf{d} using, $\mathbf{d} = \mathbf{L}\mathbf{m}$, where \mathbf{m} is a reflectivity model. The quadratic cost function,

$$S(\mathbf{m}) = \|\mathbf{d} - \mathbf{d}_{obs}\| = \|\mathbf{L}\mathbf{m} - \mathbf{d}_{obs}\|, \quad (1)$$

is formed, where \mathbf{d}_{obs} denotes the recorded data. The reflectivity model $\hat{\mathbf{m}}$ that minimizes $S(\mathbf{m})$ is given by

$$\hat{\mathbf{m}} = (\mathbf{L}'\mathbf{L})^{-1} \mathbf{L}'\mathbf{d}_{obs} = \mathbf{H}^{-1}\mathbf{m}_{mig} \quad (2)$$

where the migration operator \mathbf{L}' is the adjoint of the modelling operator \mathbf{L} , \mathbf{m}_{mig} is the migration image, and \mathbf{H} is the Hessian of $S(\mathbf{m})$, also know as resolution function, or point spread function (PSF). It is often more feasible to compute the inverse image as the solution of the linear system of equations

$$\mathbf{H}\hat{\mathbf{m}} = \mathbf{m}_{mig} \quad (3)$$

by using an iterative inversion algorithm (Valenciano et al., 2006). Equation 3 expresses the convolutional relation between the ideal reflectivity $\hat{\mathbf{m}}$ and the migrated image \mathbf{m}_{mig} . It explains how only a filtered version of the true reflectivity is recovered by migration. Being able to estimate the PSF is the key to produce high-resolution images.

How does the PSF relate to image resolution?

The PSF is the response of a point scatterer through the imaging system (Equation 3). By taking a spatial FFT of the PSF the wave number coverage is obtained. Figure 1 demonstrates the concept in a 2D constant velocity medium. It shows the space-Fourier pairs corresponding to a point scatterer after convolution (migration) with the PSF (left), and the result of WEI (right). As the velocity is constant the wavenumber coverage and image resolution is only limited by the acquisition geometry and the wavelet bandwidth. WEI substantially increases the spectral balance and spatial resolution.

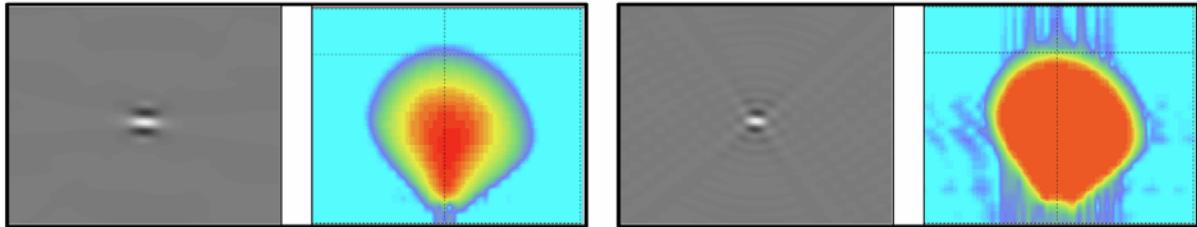


Figure 1. Space-Fourier pairs corresponding to a point scatterer after convolution (migration) with the PSF (left), and the result of WEI (right).

The Sigsbee2A model

The sediment section of the Sigsbee2A model is ideal for illustrating the effect of the inversion on the image resolution, as it has sharp contrast features due to faulting. Figure 2 (top left) shows the depth migrated image, that can be compared with the WEI result in Figure 2 (top right). The inversion shows better resolution and more balanced amplitudes than the migration. A close-up view of the space-Fourier pairs at one of the faults in Figures 2 (bottom left and right) shows the improvement in spatial resolution and wavenumber content. The wavenumber slopes corresponding to the fault and the flat reflector are clearly improved by the inversion.

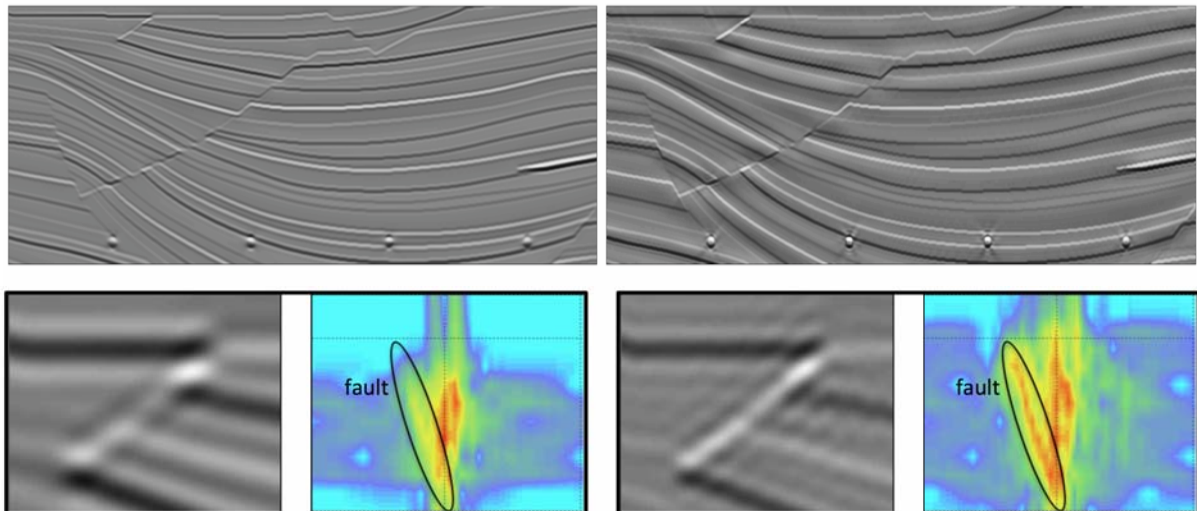


Figure 2. Depth images: migration (left), and WEI (right). The bottom figures show a close-up view of the space-Fourier pairs computed at one of the faults.

Field data from the North Sea

We applied WEI to a 3D image from the North Sea. The data was acquired with dual-sensor streamers comprised of hydrophones and vertical geophones. Although the receiver ghost was effectively removed during time processing, WEI can further improve the wavenumber content and resolution.

Figure 3 compares the VTI migration with the WEI result. Overall, the inversion shows more balanced amplitudes than the migration. Also note the reduction of the sail-line acquisition footprint in the WEI image. The reflectors gain vertical and horizontal resolution, making them more interpretable (specially the faults). The temporal spectrum comparison, after depth to time conversion, is shown in Figure 4. The WEI result displays a broader spectrum, where both low and the high frequencies are enhanced. The improvements are corroborated by the comparison of the wavenumber spectrum for both the migration and the WEI inside the red box in Figure 3. Note how the spectral coverage is greatly improved by WEI, specially the wavenumber slopes corresponding to the fault.

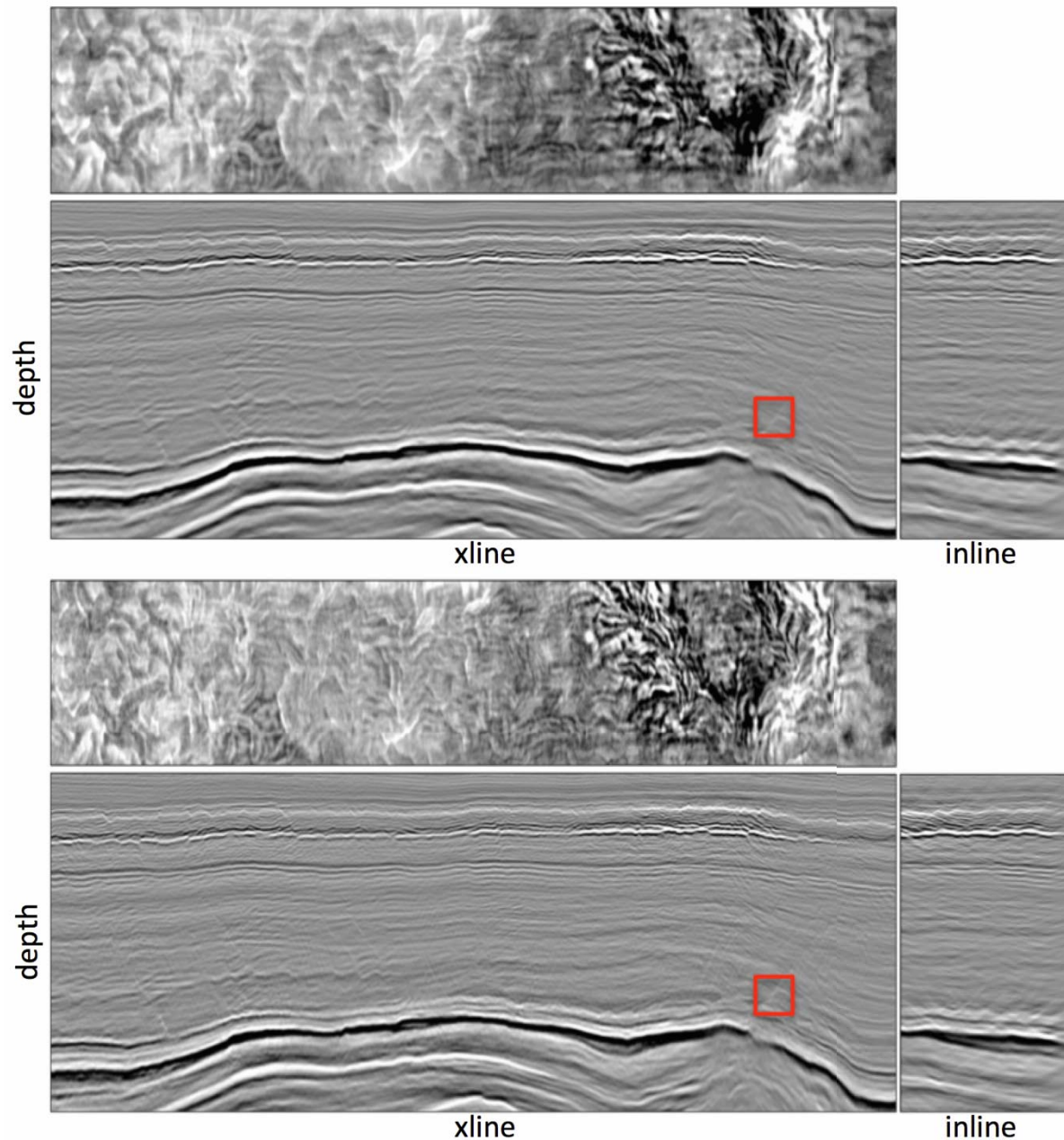


Figure 3. Depth images: migration (top), and WEI (bottom).

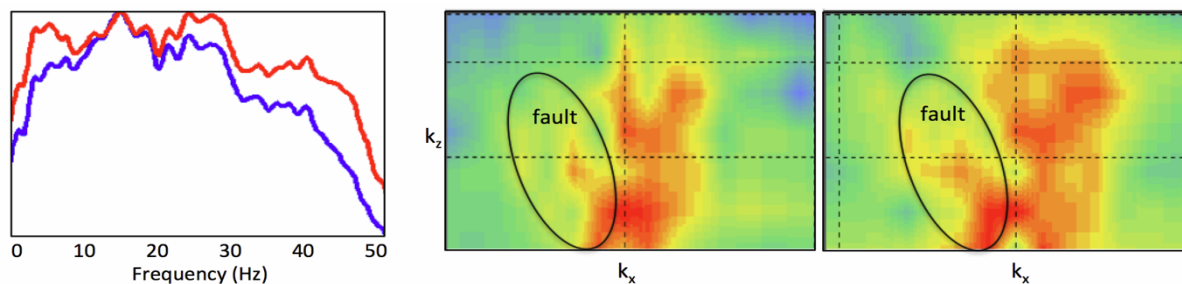


Figure 4. Temporal spectrum comparison (right) between the migration (blue) and WEI (red). Comparison of the wavenumber spectrum for both migration (centre) and WEI (left) computed in the red box in Figure 3.

Conclusions

Wave equation reflectivity inversion (WEI) is a robust technique for enhancing the wave number content and improving the resolution of depth migrated images. Synthetic data results demonstrate a clear improvement of the sharp features in the image (e.g. faults). Application of WEI to field dataset from the North Sea also corroborated this result. WEI greatly reduced the sail-line acquisition footprint and improved the spatial resolution of the migrated image. Overall, WEI provides images with broader spatial and temporal spectra with higher quality from shallow to deep in the section.

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References

- Carlson, D., Sollner, W., Tabti, H, Brox, E., and Widmaier, M., 2007, Increased resolution of seismic data from a dual sensor streamer cable: 79th Annual International Meeting, Expanded Abstracts, 994–998, Soc. Expl. Geophysics.
- Claerbout, J. F., 1992, Earth soundings analysis, processing versus inversion: Blackwell Scientific Publications.
- Gelius, L.J., I. Lecomte, and H. Tabti, 2002, Analysis of the resolution function in seismic prestack depth imaging: Geophysical Prospecting, 50, 505–515.
- Guittou, A., 2004, Amplitude and kinematic corrections of migrated images for nonunitary imaging operators: Geophysics, 69, 1017–1024.
- Hu, J., G. T. Schuster, and P. Valasek, 2001, Poststack migration deconvolution: Geophysics, 66, 939–952.
- Lecomte, I., 2008, Resolution and illumination analyses in psdm: A ray-based approach: The Leading Edge, 27, 650–663.
- Nemeth, T., C. Wu, and G. T. Schuster, 1999, Least-squares migration of incomplete reflection data: Geophysics, 64, 208–221.
- Plessix, R.E. and W. A. Mulder, 2004, Frequency-domain finite-difference amplitude-preserving migration: Geophysical Journal International, 157, 975–987.
- Prucha, M. and B. Biondi, 2002, Subsalt event regularization with steering filters: 72th Annual International Meeting, Expanded Abstracts, 922–925, Soc. Expl. Geophysics.
- Rickett, J.E., 2003, Illumination-based normalization for wave-equation depth migration: Geophysics, 68, 1371–1379.
- Valenciano, A. A., B. Biondi, and A. Guittou, 2006, Target-oriented wave-equation inversion: Geophysics, 71, A35–A38.