Multi-Parameter FWI - Long-Wavelength Updates and Leakage Reduction in Acoustic Anisotropic Media

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Summary

Multi-parameter Full Waveform Inversion (FWI) suffers from crosstalk (leakage) between the different medium properties. We propose a practical solution that minimizes the leakage and retrieves the long-wavelength features of the model. Our algorithm is based on a regularization of the objective function and a specific parameterization of an acoustic Transversely Isotropic (TI) medium: vertical velocity, and Thomsen parameters epsilon and delta. From this chosen parameterization the resulting vertical velocity sensitivity kernel produces long wavelength updates. For the inversion of epsilon, the long-wavelength updates are enforced by the application of total variation regularization (TV). We show, by using synthetic data, that TV regularization significantly reduces the leakage of the vertical velocity in the epsilon model during the inversion. Finally, we show an application on field data from the Gulf of Mexico where the flatness of the common image gathers is significantly improved.

Introduction

A successful multi-parameter inversion requires a careful choice of the parameterization to resolve the different medium properties and to avoid crosstalk (leakage) (e.g., Plessix and Cao, 2011; Alkhalifah and Plessix, 2014; Cheng et al., 2015). Besides the use of a favorable parameterization, practitioners often use specific data selection strategies based on theoretical or practical considerations. While working with field data they also take into account that different degree of accuracy or resolution that is needed from the estimated medium properties in order to fit the data.

If the inversion is conducted in an acoustic Transversely Isotropic medium with vertical symmetry axis (VTI) there are several medium parameter choices available. Alkhalifah and Plessix (2014) analysed the radiation patterns, for different parameterizations; the aim of their study was to define how well the anisotropic parameters could be resolved from the different wave propagation directions. They confirmed that there is not enough information to invert for Thomsen parameter δ from surface seismic data. Thus, the use of independent borehole data to estimate the δ field is required. A conclusion obtained by other authors (e.g., Plessix and Cao, 2011, Debens et al., 2015), is that in order to match the kinematics of the propagating waves at large offsets, it is sufficient to produce a low-resolution ε field. In addition, practical applications call for different data selection strategies that could be used to reduce the leakage of the different medium properties. Cheng et al. (2016) concluded that the diving waves sense in a similar way both vertical velocity v_z and ε. They recommend the inclusion of reflected events in the FWI when v_z is inverted in order to provide independent information and reduce the leakage in the two-parameter updates. Another alternative is to filter the gradient by wavenumber content (Alkhalifah, 2015); this has the potential to reduce the leakage of the different model parameters.

Here we describe the sensitivity kernels for the vertical velocity and epsilon computed for a parameterization consisting of v_z, and the Thomsen parameters ε and δ. Assuming that δ is held constant during the inversion, the v_z sensitivity kernel has a similar form as that for the isotropic case proposed by Ramos-Martinez et al. (2016). Naturally, this kernel allows for long-wavelength model updates. For estimation of ε, we retrieve the long-wavelength updates with the help of Total Variation (TV) regularization (Guo and de Hoop, 2013). The regularization also helps to minimize the leakage of the v_z field on ε. As shown from synthetic data, this leakage occurs in the ε field inversion even when the vertical velocity is correct. First we show the forms of the sensitivity kernels for v_z and ε kernels in the context of the pseudo-analytical extrapolator (Etgen, and Brandsberg-Dahl, 2009). Then, by using a synthetic example, we validate the ε kernel and the role of the regularization in the minimization of the leakage between parameters. Finally, we show a successful application in a field dual-sensor dataset from the Gulf of Mexico, performing a recursive flow consisting of the independent inversion of v_z from reflections and diving waves, and then the inversion of ε from diving waves only.

Theory

Our FWI solves the acoustic wave equation by using the pseudo-analytical method (Etgen, and Brandsberg-Dahl, 2009). Their derivation departs from the VTI dispersion relation proposed by Harlan and Lazear (1998, personal communication)

$$\omega^2 = v_z^2 k_z^2 + v_h^2\left(k_x^2 + k_y^2\right) + \left(v_{nmo}^2 - v_h^2\right)\left(k_x^2 + k_y^2\right)k_z^2 / \left(k_x^2 + k_y^2 + k_z^2\right) \quad (1)$$

where ω is the angular frequency, v_z, v_h and v_{nmo} are the vertical, horizontal and NMO velocities, k_i are the wavenumber vector components along the space coordinates $\mathbf{x}$. After transforming equation (1) to space-time domain, and solving the time derivative using a second-order finite-difference approximation, we obtain the following time-stepping scheme:

$$\begin{aligned} S(\mathbf{x},t+\Delta t) &= 2S(\mathbf{x},t) - S(\mathbf{x},t-\Delta t) + \Delta t^2 v_z^2 FT^{-1}\left[f_z(k_z)S(\mathbf{k},t)\right] + \\ &+ \Delta t^2 v_h^2 FT^{-1}\left[f_h(k_x,k_y)S(\mathbf{k},t)\right] + \Delta t^2\left(v_{nmo}^2(\mathbf{x}) - v_h^2(\mathbf{x})\right)FT^{-1}\left[f_n(k_x,k_y,k_z)S(\mathbf{k},t)\right] + s_f \end{aligned} \quad (2)$$

where S is the forward-propagated wavefield, the point-source term with source wavelet $w(t)$ is $s_f = \delta(\mathbf{x}-\mathbf{x}_s)w(t)$, and FT^{-1} stands for inverse Fourier Transform. The differential operators f_i are combinations of the normalized pseudo-Laplacian operator (Chiu and Stoffa, 2011), which correct for the inaccuracy produced by of the second-order finite-difference approximation to the time derivative with time step Δt. For example:

$$f_z(k_z) = \frac{2\cos(v_v|\mathbf{k}|\Delta t)-2}{\Delta t^2 v_z^2 |\mathbf{k}|^2}\left(k_z^2\right); \quad f_h(k_x,k_y) = \frac{2\cos(v_h|\mathbf{k}|\Delta t)-2}{\Delta t^2 v_h^2 |\mathbf{k}|^2}\left(k_x^2+k_y^2\right).$$

The adjoint-state equation corresponding to the state equation (2) has the form

$$R(\mathbf{x},t+\Delta t) = 2R(\mathbf{x},t) - R(\mathbf{x},t-\Delta t) + \Delta t\; v_z^2 FT^{-1}\left[f_z(k_z)R(\mathbf{k},t)\right] + \Delta t^2 v_h^2 FT^{-1}\left[f_h(k_x,k_y)FT\{(1+2\varepsilon)R(\mathbf{x},t)\}\right] +$$
$$+\Delta t^2\left(v_{nmo}^2(\mathbf{x}) - v_h^2(\mathbf{x})\right)FT^{-1}\left[f_n(k_x,k_y,k_z)FT\{2(\varepsilon-\delta)R(\mathbf{x},t)\}\right] + \bar{s}_f \qquad (3)$$

where R is the back-propagated wavefield, ε and δ are the Thomsen parameters. The adjoint source term is the residual wavefield $\bar{s} = S(\mathbf{x}_R,t) - d(\mathbf{x}_R,t)$. For a parameterization consisting of v_z, acoustic impedance (computed from v_z), ε and δ, and assuming that δ is constant during the inversion, the gradients for v_z and ε have the following form

$$G_{v_z}(\mathbf{x}) = \frac{1}{2A_{v_z}(\mathbf{x})}\left[\int_t \left[-W_1(\mathbf{x},t)\nabla S(\mathbf{x},t)\cdot\nabla R(\mathbf{x},T-t) + W_2(\mathbf{x},t)\frac{1}{v_z^2(\mathbf{x})}\frac{\partial S(\mathbf{x},t)}{\partial t}\cdot\frac{\partial R(\mathbf{x},T-t)}{\partial t}\right]dt\right] \qquad (4)$$

$$G_\varepsilon(\mathbf{x}) = -\frac{2}{A_\varepsilon(\mathbf{x})}\int\left[FT^{-1}\{f_h(k_x,k_y)S(\mathbf{k},t) - (f_n(k_x,k_y,k_z)S(\mathbf{k},t)\}\right]\cdot R(\mathbf{x},T-t)dt\,, \qquad (5)$$

where A_{v_z} and A_ε are source illumination terms, and W_i are dynamic weights designed to optimally suppress the high wavenumber components in the gradient (Ramos-Martínez et al., 2016).

For mathematical convenience we have make the derivations in a VTI medium. In general applications one needs to handle a Transversely Isotropic medium with tilted symmetry axis (TTI). The previous derivations can be extended for the TTI case by a rotation of the spatial wavemumbers to match the tilted symmetry axes. The vertical velocity gradient (equation 4) is designed to produce long wavelength updates. On the other hand, the epsilon gradient in equation 5 is form by crosscorrelation of a modified adjoint source with the data residuals. In order to enforce long-wavelength updates with this gradient we applied a total variation regularization (Guo and de Hoop, 2013). Our procedure makes use of the split Bregman iterations (Goldstein and Osher, 2009), an effective algorithm for solving the L^1 optimization problems. The result is a computationally efficient and accurate implementation.

Examples

We construct a simple synthetic example to illustrate the performance of the $\boldsymbol{\varepsilon}$ gradient (eq. 5) and the significance of aggressive TV regularization to reduce the leakage between the different inverted parameters. Figure 1a shows the $\boldsymbol{v_z}$ model, which is a modified version of the sediment portion of the SEAM model, in which we include a velocity anomaly. Figure 1b shows the difference between the true and the starting epsilon fields. We invert for $\boldsymbol{\varepsilon}$ assuming that velocity field is exact. Figures 1c and 1d, shows the final $\boldsymbol{\varepsilon}$ updates without and with regularization, respectively. As observed, the imprint of the velocity anomaly is significantly reduced when aggressive regularization is used.

Finally, we show a field example corresponding to dual-sensor data acquired in the De Soto Canyon area in the Gulf of Mexico, with a maximum offset of 12 km. Figure 2a shows the starting vertical velocity model. Epsilon and delta initial models are zero in the water column and constant from the water bottom with values of 0.08 and 0.04, respectively. We use a maximum frequency of 7 Hz for the velocity inversion, and as expected, only the far offset are improved after epsilon estimation. In the second stage, the gathers are further improved. Figure 2b and Figure 2c show the final vertical velocity and epsilon models after two stages of inversion in cascade. Figures 3a and 3b show Kirchhoff image gathers across the model before and after the inversion of both vertical velocity and epsilon, in which is evident the improvement of the gathers after inversion.

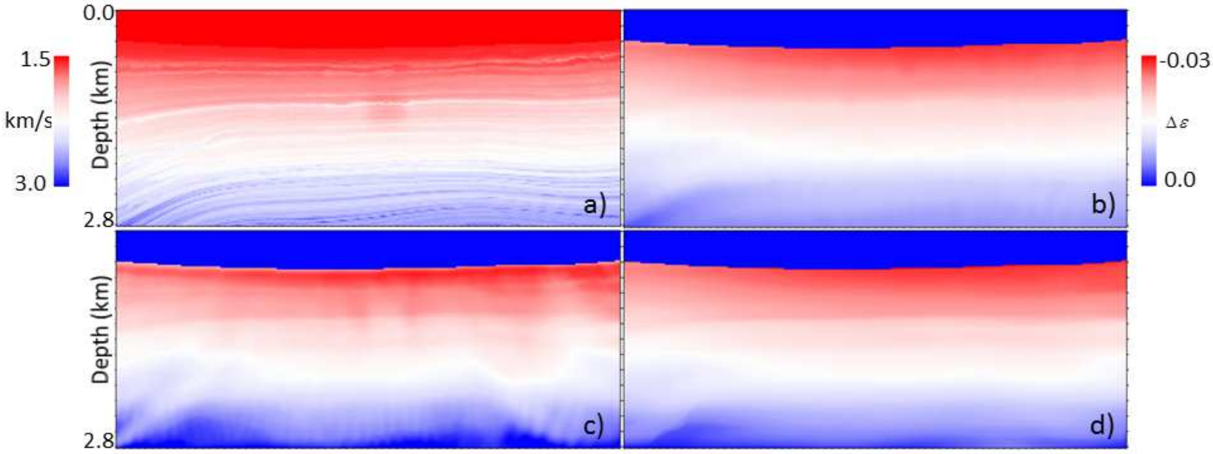

Figure 1. a) Vertical velocity and b) difference between the true and starting epsilon. Epsilon updates after inversion without (c) and with (d) TV regularization.

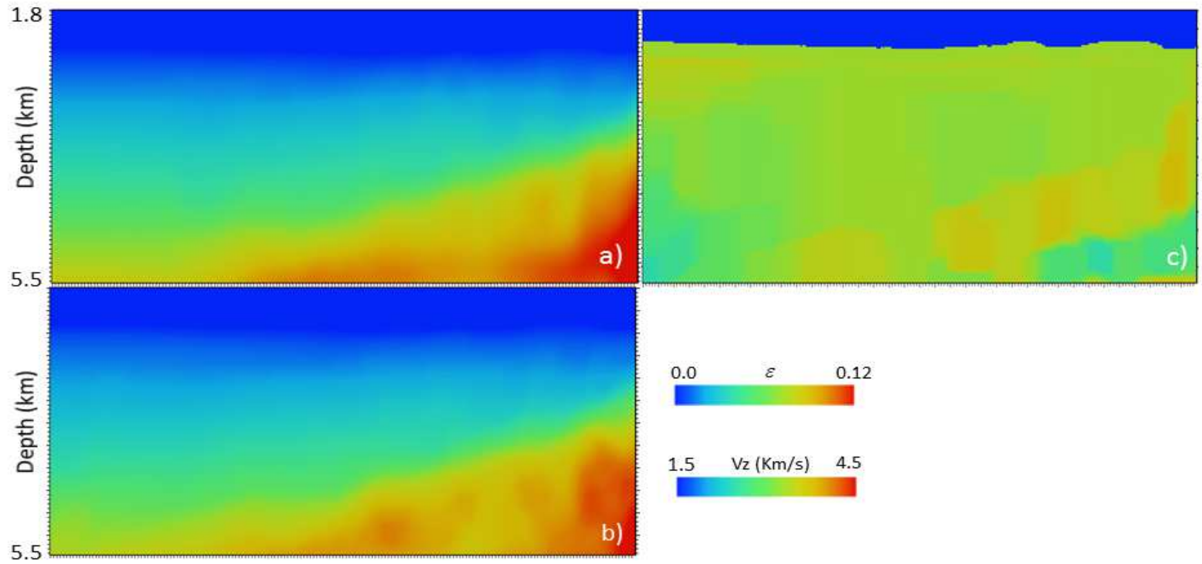

Figure 2. a) Starting vertical velocity model for the Gulf of Mexico field data example; epsilon and delta models are constant from the water bottom and equal to 0,08 and 0.04, respectively. Inverted b) vertical velocity and c) epsilon models after two stages of sequential inversion.

Conclusions

We introduce a practical FWI approach to retrieve the long wavelength updates in an acoustic anisotropic medium with transverse isotropy. We use a parameterization consisting of vertical velocity, epsilon and delta, to obtain the sensitivity kernels for vertical velocity and epsilon. First, we updated the vertical velocity from a long-wavelength gradient, which has a similar form than that derived from the isotropic velocity sensitivity kernel that is able to suppress migration isochrones. Then, long-wavelength epsilon updates are obtained by TV regularization, which significantly reduces the leakage of the velocity imprint in the epsilon field. We show a successful application of the approach to a dual-sensor dataset acquired in the Gulf of Mexico, which is validated by the improvement of the flatness of the image gathers using the inverted models after two stages of cascaded inversions of vertical inversion and epsilon.

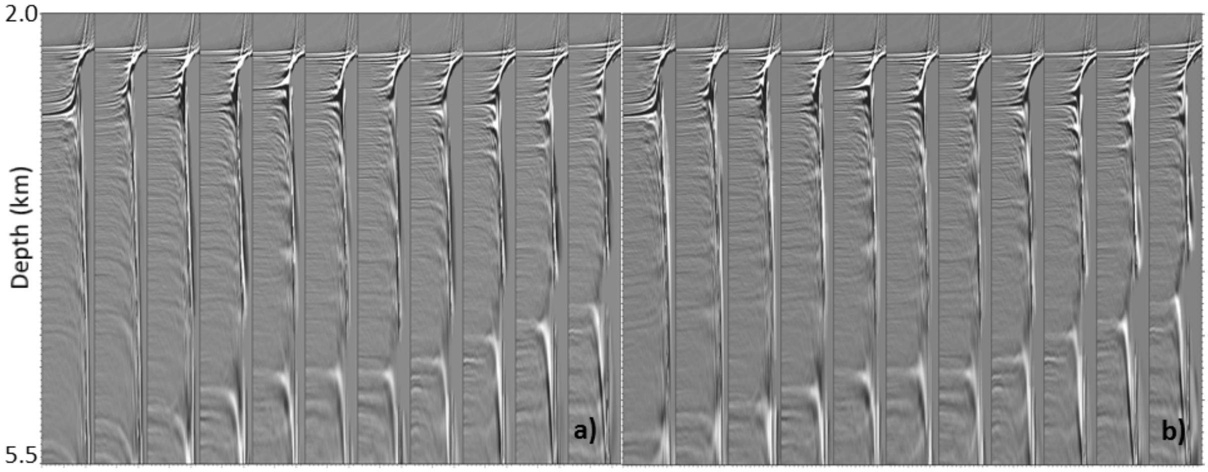

Figure 3. Kirchhoff image gathers for the a) starting and b) inverted vertical velocity and epsilon models corresponding to the field data example of Gulf of Mexico.

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