Compensating visco-acoustic effects in anisotropic resverse-time migration

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SUMMARY

Anelastic properties of the earth cause frequency dependent energy attenuation and phase distortion in seismic wave propagation. It is preferred that these unwanted effects be corrected in a prestack depth migration. Zhang et al (2010) introduced a visco-acoustic wave equation in the time domain for isotropic media. This paper extends the visco-acoustic wave equation for anisotropic case, and develops visco-acoustic reverse time migration algorithm for VTI medium. To validate the proposed wave equation, wave propagation is simulated on a homogeneous viscous VTI medium using a finite difference method. The wavefield snapshot shows predicted frequency dependent attenuation and dispersion. Synthetic and field data examples are also given.

INTRODUCTION

Some shallow geologic features such as gas chimneys can heavily attenuate seismic P-waves. A seismic image below a gas anomaly can be severely deteriorated by a decrease of overall amplitude, a loss of high-frequency energy, and also a distortion of the seismic phase. Dramatic lateral variations of shallow attenuation properties associated with gas can generate strong footprints on deep seismic images, destroying AVO/AVA effects and making the interpretation more difficult. It is therefore very important to compensate these attenuation effects in order to improve our final seismic products (Yu et al., 2002).

Early work to compensate for seismic attenuation was performed in the unmigrated data domain by an inverse Q-filter (Bickel and Natarajan, 1985; Hargreaves and Calvert, 1991, Wang, 2006). These methods were based on a one-dimensional backward propagation and cannot correctly handle real geological complexity. Since anelastic attenuation and dispersion occur during the wave propagation, it is natural to correct them in a prestack depth migration (Zhang et al., 2010).

Much effort has been put forth in developing an inverse Q-migration using one-way wave equation migration (Dai and West, 1994; Yu et al., 2002). Zhang et al (2010) derived a visco-acoustic wave equation in the time domain and applied it in a prestack reverse time migration to compensate for the anelstic effects in the seismic data. The equation is applicable to isotropic media.

Recently, anisotropic reverse time migration is considered a standard tool for subsalt imaging. To compensate for seismic attenuation in anisotropic media, the corresponding visco-acoustic wave equation has to be derived. In this paper, we extend Zhang et al's equation to anisotropic media. The equation is numerically solved to show that it can properly simulate for the frequency dependent absorption and dispersion effects. Later, we show synthetic migration example and field data example.

METHOD

The dispersion relation of a linear visco-acoustic medium can be written by

$$k = \frac{\omega}{v} - i\tan(\frac{\pi\gamma}{2})\frac{|\omega|}{v} \tag{1}$$

where k and ω are the spatial wavenumber and angular temporal frequency, respectively. The frequency dependent velocity v is given by

$$v = v_0|\frac{\omega}{\omega_0}|^\gamma \tag{2}$$

where v_0 is the reference velocity at the reference frequency ω_0 and the dimensionless quantity γ is defined by

$$\gamma = \frac{1}{\pi}\tan^{-1}(\frac{1}{Q}) \approx \frac{1}{\pi Q}, \tag{3}$$

with Q being the quality factor (Kjartansson, 1979).

Assuming the attenuation ($1/Q$) is small, Zhang et al (2010) derived the time domain equivalent of the above as

$$p_{tt} + \frac{\Phi}{Q}p_t + \Phi^2 p = 0 \tag{4}$$

where the subscript t indicating the partial derivative operation with respect to time of the pressure field p, and Φ is a pseudo-differential operator in the space domain defined by

$$\Phi = \left(\frac{\sqrt{-v_0^2\nabla^2}}{\omega_0^\gamma}\right)^{\frac{1}{1-\gamma}}. \tag{5}$$

Equation (4) is a visco-acoustic wave equation for an isotropic media. The second term contributes wavefield attenuation. Therefore removing the lossy term as

$$p_{tt} + \Phi^2 p = 0 \tag{6}$$

we get frequency dependent velocity dispersion only (dispersive-acoustic wave equation). On the contrary, if we preserve the lossy term but modify the dispersive term as

$$p_{tt} + \frac{\Phi}{Q}p_t - v^2\nabla^2 p = 0 \tag{7}$$

we get attenuation only (lossy-acoustic wave equation). Chen and Holm (2004) suggested fractional Laplacian time-space wave equation for lossy media exhibiting arbitrary frequency power-law dependency. Equation (7) corresponds to their Equation (21) with the power constant $y = 1$.

Figure 1 shows four wavefield snapshots of a two-dimensional homogeneous isotropic model computed by a finite-difference method. The computing parameters are 3000 m/s velocity, Q value of 20, 10 m grid spacing, 20 Hz peak frequency Ricker

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wavelet source, and 15 Hz reference frequency, The four snapshots represent the wavefield of (a) acoustic wave equation, (b) dispersive-acoustic wave equation, (c) lossy-acoustic wave equation, and (d) visco-acoustic wave equation, respectively. Comparing the snapshots we notice that the changes from (a) to (b) and from (c) to (d) are not so significant than the changes from (a) to (c) and (b) to (d). This means that dispersion has less impact than the attenuation.

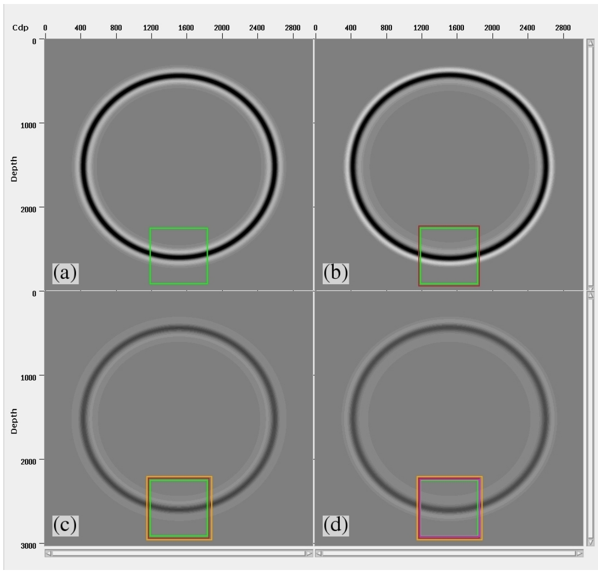

Figure 1: Wavefield snapshots from four different acoustic wave equations: (a) acoustic, (b) dispersive-acoustic, (c) lossy-acoustic, and (d) visco-acoustic.

Figure 2 shows the frequency spectra of Figure 1, where the analysis windows are indicated by colored rectangles. The more significant feature of the plot is the frequency dependent wavefield attenuation.

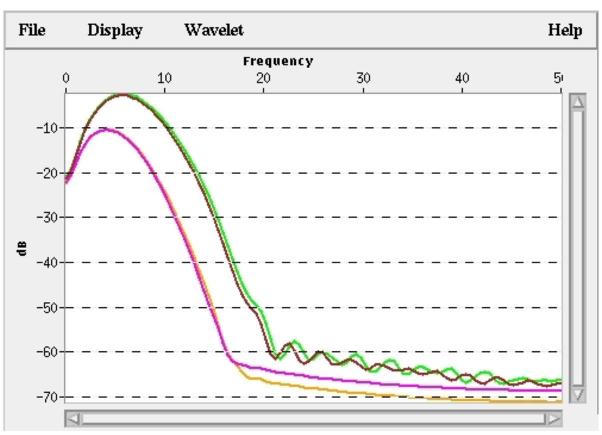

Figure 2: Frequency spectra of wavefields shown in Figure 1. The analysis windows are indicated by colored rectangles.

A pseudo-acoustic TI wave equation based on Alkhalifah's zero shear wave vertical velocity approximation is given by Fletcher et al (2009)

$$
\begin{aligned}
p_{tt} - v_x^2 H_2 p - v_z^2 H_1 q &= 0 \\
q_{tt} - v_n^2 H_2 p - v_z^2 H_1 q &= 0.
\end{aligned} \tag{8}
$$

where v_z is the velocity along the symmetry axis, v_x is the velocity in the symmetry plane, and v_n is the normal moveout velocity. The three velocities have following relationship.

$$
\begin{aligned}
v_x &= v_z\sqrt{1+2\varepsilon} \\
v_n &= v_z\sqrt{1+2\delta}
\end{aligned}
$$

where ε and δ are anisotropic parameters defined by Thomsen (1986). Here the differential operators, H_1 and H_2 are given by

$$
\begin{aligned}
H_1 &= \sin^2\theta\cos^2\phi\,\partial_{xx} + \sin^2\theta\sin^2\phi\,\partial_{yy} + \cos^2\theta\,\partial_{zz} \\
&+ \sin^2\theta\sin 2\phi\,\partial_{xy} + \sin 2\theta\sin\phi\,\partial_{yz} + \sin 2\theta\cos\phi\,\partial_{xz} \\
H_2 &= \partial_{xx} + \partial_{yy} + \partial_{zz} - H_1
\end{aligned}
$$

where θ is the dip angle and ϕ is the azimuth.

Assuming $\gamma << 1$, we modify Equation (8) to include the attenuation and dispersion so that it can be a visco-acoustic TI equation as follow:

$$
\begin{aligned}
p_{tt} + \frac{\Phi_x}{Q} p_t + \Phi_x^2 p + \Phi_z^2 q &= 0 \\
q_{tt} + \frac{\Phi_z}{Q} q_t + \Phi_n^2 p + \Phi_z^2 q &= 0.
\end{aligned} \tag{9}
$$

The three psedo-differential operators Φ_x, Φ_z, and Φ_n are defined by

$$
\begin{aligned}
\Phi_x &= \left(\frac{\sqrt{-v_{x0}^2 H_2}}{\omega_0^\gamma}\right)^{\frac{1}{1-\gamma}} \\
\Phi_z &= \left(\frac{\sqrt{-v_{z0}^2 H_1}}{\omega_0^\gamma}\right)^{\frac{1}{1-\gamma}} \\
\Phi_n &= \left(\frac{\sqrt{-v_{n0}^2 H_2}}{\omega_0^\gamma}\right)^{\frac{1}{1-\gamma}}
\end{aligned} \tag{10}
$$

where v_{x0}, v_{z0} and v_{n0} are reference velocities of v_x, v_z, and v_n respectively.

As described in isotropic case, the second terms of Equation (9) control attenuation while the third and fourth terms affect dispersion. In other words, the dispersive TI equation is

$$
\begin{aligned}
p_{tt} + \Phi_x^2 p + \Phi_z^2 q &= 0 \\
q_{tt} + \Phi_n^2 p + \Phi_z^2 q &= 0.
\end{aligned} \tag{11}
$$

and the lossy TI equation is

$$
\begin{aligned}
p_{tt} + \frac{\Phi_x}{Q} p_t - v_x^2 H_2 p - v_z^2 H_1 q &= 0 \\
q_{tt} + \frac{\Phi_z}{Q} q_t - v_n^2 H_2 p - v_z^2 H_1 q &= 0.
\end{aligned} \tag{12}
$$

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Figure 3 shows four wavefield snapshots of a two-dimensional homogeneous visco-acoustic TI model computed by a finite-difference method. The computing parameters are v = 3000 m/s, ε = 0.24, δ = 0.1, Q = 20, 10 m grid spacing, 20 Hz peak frequency Ricker wavelet source, and 15 Hz reference frequency. The dip angle θ and azimuth ϕ are zero. The four snapshots represent the wavefield of (a) TI wave equation, (b) dispersive TI wave equation, (c) lossy TI wave equation, and (d) visco-acoustic TI wave equation, respectively. Comparing the snapshots we notice that the changes from (a) to (b) and from (c) to (d) are not so significant than the changes from (a) to (c) and (b) to (d). This means that dispersion has less impact than the attenuation.

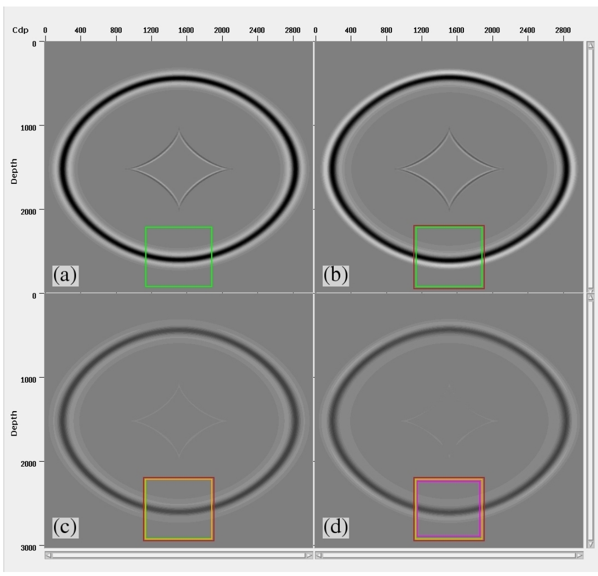

Figure 3: Wavefield snapshots from four different TI wave equations: (a) TI, (b) dispersive TI, (c) lossy TI, and (d) visco-acoustic TI.

Figure 4 shows the frequency spectra of Figure 1, where the analysis windows are indicated by rectangles. Again in this example we see there is significant frequency dependent wavefield attenuation.

During the migration, velocity dependent dispersion changes the reflection depth. This is an unwanted effect. We don't want to change the reflection position, but only to compensate for the attenuation. Therefore the lossy TI equation is more useful than the visco-acoustic TI equation. To compensate the attenuation, the sign of the second term of Equation (12) should be changed, i.e.,

$$p_{tt} - \frac{\Phi_x}{Q} p_t - v_x^2 H_2 p - v_z^2 H_1 q = 0 \qquad (13)$$
$$q_{tt} - \frac{\Phi_z}{Q} q_t - v_n^2 H_2 p - v_z^2 H_1 q = 0.$$

The equation is applied both on source wavefield and on receiver wavefield. This amplifies high frequency component and results instability. Dai and West (1994) suggested adjusting the working frequency band and/or modifying the attenuation coefficient in their inverse Q migation. We suggest limited high frequency filtering, and only apply the high frequency filtering on (p_t) which is the wavefield attenuation term related to viscosity, and without touching the wavefield propogation term.

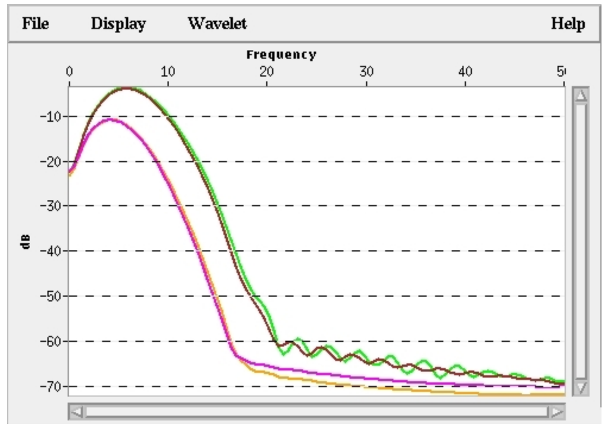

Figure 4: Frequency spectra of wavefields shown in Figure 3.

EXAMPLES

Figure 5a shows a 12 x 6 km two-dimensional earth model of three horizontal layers and a gas pocket whose physical properties are listed in Table 1. A total of 200 synthetic shots were computed by a finite-difference method based on Equation (9). The receiver offsets are 0 to 2980 m incrementing 20 m. The shot spacing was 50 m.

layer	v (m/s)	ε	δ	Q
1	1500	0	0	500
2	2000	0.3	0.2	100
3	2500	0.2	0.1	1000
gas pocket	1600	0.5	0.25	20

Table 1: Physical properties of the model in Figure 5a.

The synthetic data was migrated without Q compensation. Figure 5b shows the result. Because of the viscosity, the regular RTM produces a distorted image at the bottom of the gas pocket and at the third layer right below the gas pocket, with dimming of amplitude of distorted phase as compared with the other parts of image without much attenuation effects. Figure 5c is Q-RTM result using Equation (13) Clearly our Q-RTM is able to compensate for the attenuation effect, with more uniform amplitude and corrected phase (zero-phase everywhere).

Figure 6a shows a section of VTI reverse time migration image of Gulf of Mexico data. The section contains irregular reflectors at depth 3000 meters causing amplitude imbalance to the deeper part. Because the cause of amplitude attenuation is diffraction rather than viscosity, we tried to recover the attenuation only. A Q model was determined by analyzing the

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amplitude and frequency content of the first arrivals contributing the flat reflection at 4 km. Figure 6b is the zoomed version of Figure 6a and Figure 6c is the result of Q compensated reverse time migration using lossy TI equation. The improved areas are highlighted by yellow circles.

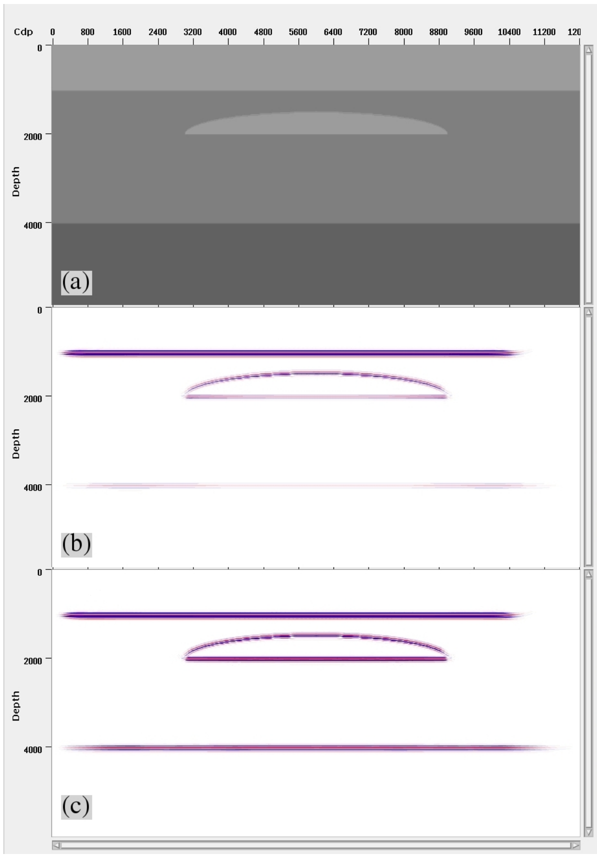

Figure 5: A synthetic Q RTM example. (a) two-dimensional gas pocket bearing earth model, (b) Regular RTM image without Q compensation, (c) New RTM with Q compensation.

CONCLUSION

We have proposed a new visco-acoustic TI propagator for anisotropic media. The equation is intuitively extended from the visco-acoustic equation for isotropic media introduced by Zhang et al (2010). Numerical tests on homogenous two dimensional model show that the proposed anisotropic visco-acoutic equation gives frequency dependent attenuation and velocity dispersion as in the isotropic visco-acoustic equation.

Surface seismic data is attenuated both from downgoing and upgoing paths. To compensate for the attenuation, the lossy term of the visco-acoustic equation is negated. However, the resulting wave equation is unstable as in other inverse Q migrations. We workaround the instability by a limited high-cut filtering technique, i.e., applying the high-cut filter on the lossy term only.

A synthetic example of a gas pocket model shows that Q-RTM produces better structural definitions and more balanced amplitude beneath the gas pocket. A field data example from the Gulf of Mexico is given.

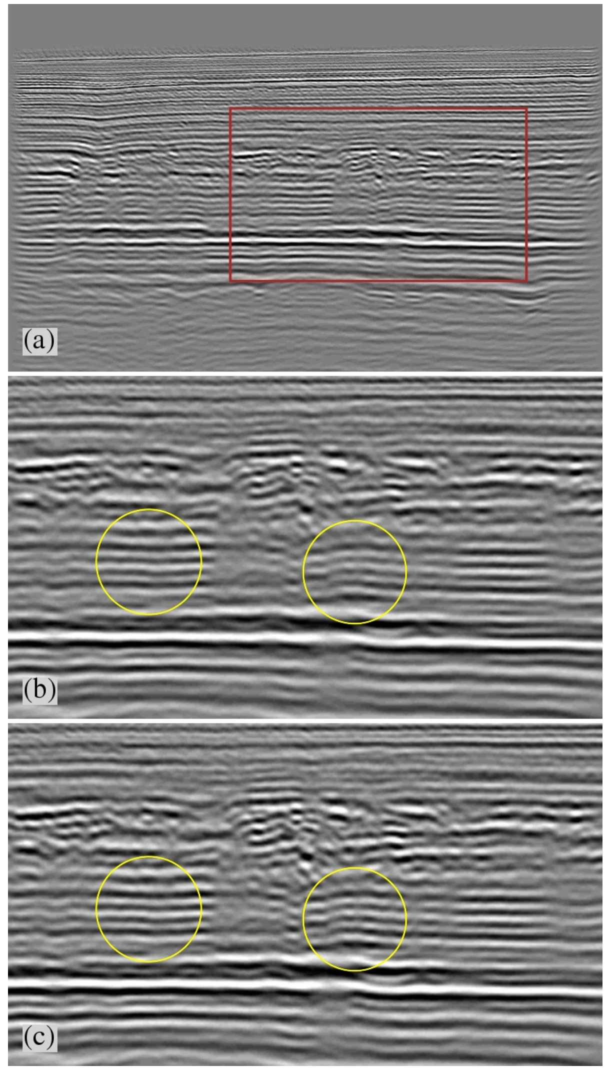

Figure 6: A VTI migration example. (a) migration without Q, (b) zoomed migration of (a), (c) zoomed migration with Q compensation. The improved areas are highlighted by yellow circles.

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